

## NON-LINEAR DYNAMIC SOIL RESPONSE UNDERNEATH A VERTICAL BREAKWATER SUBJECTED TO IMPULSIVE SEA WAVE ACTIONS

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**Summary** *A theoretical model of the soil-water-structure interaction involved in a breakwater structure subjected to sea wave actions is presented. The model includes i) soil skeleton-pore fluid interaction governed by the  $u - p_w$  Generalized Biot formulation [1] including dynamic effects, ii) non-linear soil elastoplastic behaviour described by a novel Generalized Plasticity model [2] coupled with a conservative hyperelastic formulation for the dependence of the elastic stiffness on the stress [3], iii) coupling between the caisson and foundation through a non-linear contact with geometrical compatible formulation incorporating frictional behaviour.*

*The numerical solution of the settled governing equations has been fully developed through the Finite Element Method. Furthermore, a program called ADÍNDICA has been created in M Matlab language. ADÍNDICA is a Spanish acronym for “Caisson Breakwater Dynamic Analysis”. Related numerical analyses are developed with reference to precise boundary value problems of specific physical nature. ADÍNDICA code has been able to reproduce adequately the principal characteristics of the caisson oscillations and instantaneous pore pressure generation relation deduced experimentally. Moreover, ADÍNDICA has been able to reproduce satisfactorily the accumulative settlement behaviour of a vertical breakwater structure subjected to series of sea wave impacts including the correlation between accumulated settlements and residual pore pressure.*

### 1 INTRODUCTION

The design of the foundation of marine structures presents a series of difficulties due to the complexity of the forces exerted over the structure, derived from the dynamic swell action and transmitted to the seabed through a complex foundation-structure interaction, as well as the nonlinear soil behaviour, where there is a coupling between solid skeleton and pore water.

These difficulties make the dynamics associated with a seabed underlying a vertical breakwater a uniquely complex task. It seems that the phenomena involved in these dynamics cannot be reproduced with a single model. Therefore it is necessary to couple a series of

models to adequately reproduce each of the determinant aspects concerned, for which the use of numerical techniques becomes indispensable.

Although the phenomenon of wave-induced seabed instability has received great attention among coastal geotechnical engineers since the 80s of last centuries, most of the developed researches [4, 5] have modelled the seabed soil skeleton-pore fluid interaction through the pseudostatic Biot Formulation. This theory, even if it is the base of most of subsequent developments, does not include dynamic terms. However several researches [6, 7] have shown the significant relation of these terms with the wave induced effective stress development.

Most soil models used in the investigations of sea floor dynamics have been limited to the poroelastic model. Only a few contributions [8] have incorporated advanced constitutive relations that are able to represent properly the features of soil response under cyclic loading. This is a prominent aspect within any model proposed to analyze the geomechanical behaviour associated with a breakwater foundation as is needed to investigate the possible degradation process, i.e. the change of the strength and stiffness of the soil with time, mainly due to repetitive loading. The theoretical model for the soil-water-structure interaction presented in this paper includes an advanced sand constitutive model sensitive to cyclic loads.

The caisson-rubble mound interaction phenomenon, responsible of the principal loads transmitted to the foundation, has been investigated mostly through elastic Mass-Spring-Dashpot models [9, 10] where the caisson structure is modelled as a point mass. Therefore, these models are not able to analyze the different interface strain-stress states involved in this contact surface. Few researches have employed frictional contact mechanics of deformable bodies to represent this interaction phenomenon not analyzing geomechanical implication.

In the next chapter the proposed theoretical model for the soil-water-structure interaction involved in a breakwater structure subjected to sea wave actions is presented. Afterwards the Finite Element numerical solution is outlined leading to some related numerical analyses with reference to precise boundary value problems of specific physical nature in order to justify the theoretical model and its numerical approach.

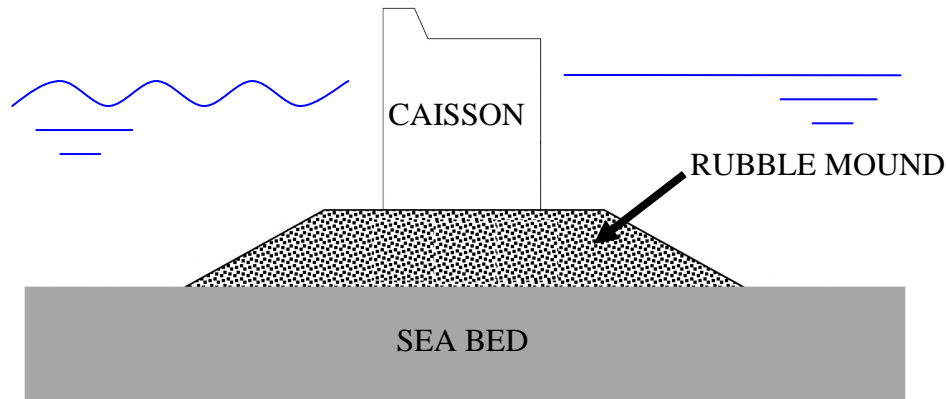
The sign convention of compressive stresses and strains negative while compressive pore pressure positive is assumed.

## 2 SOIL-WATER-STRUCTURE INTERACTION THEORETICAL MODEL.

The soil-water-breakwater interaction has been modelled coupling different physical systems, therefore independent solution of each system being impossible without simultaneous solution of the others.

The physical systems involved in the soil-water-breakwater interaction analysis are the caisson, the rubble mound and the sea bed (Figure 1). The coupling among these three occurs on domain interface via the boundary conditions imposed there. The rubble mound and the sea bed are already coupled media, where skeleton-pore fluid interaction exists, and the coupling occurs through the governing partial differential equations describing each physical phase.

Sea waves are not modelled as a proper physical system in the proposed theoretical model, representing the sea wave actions exerted over the structure as boundary conditions.

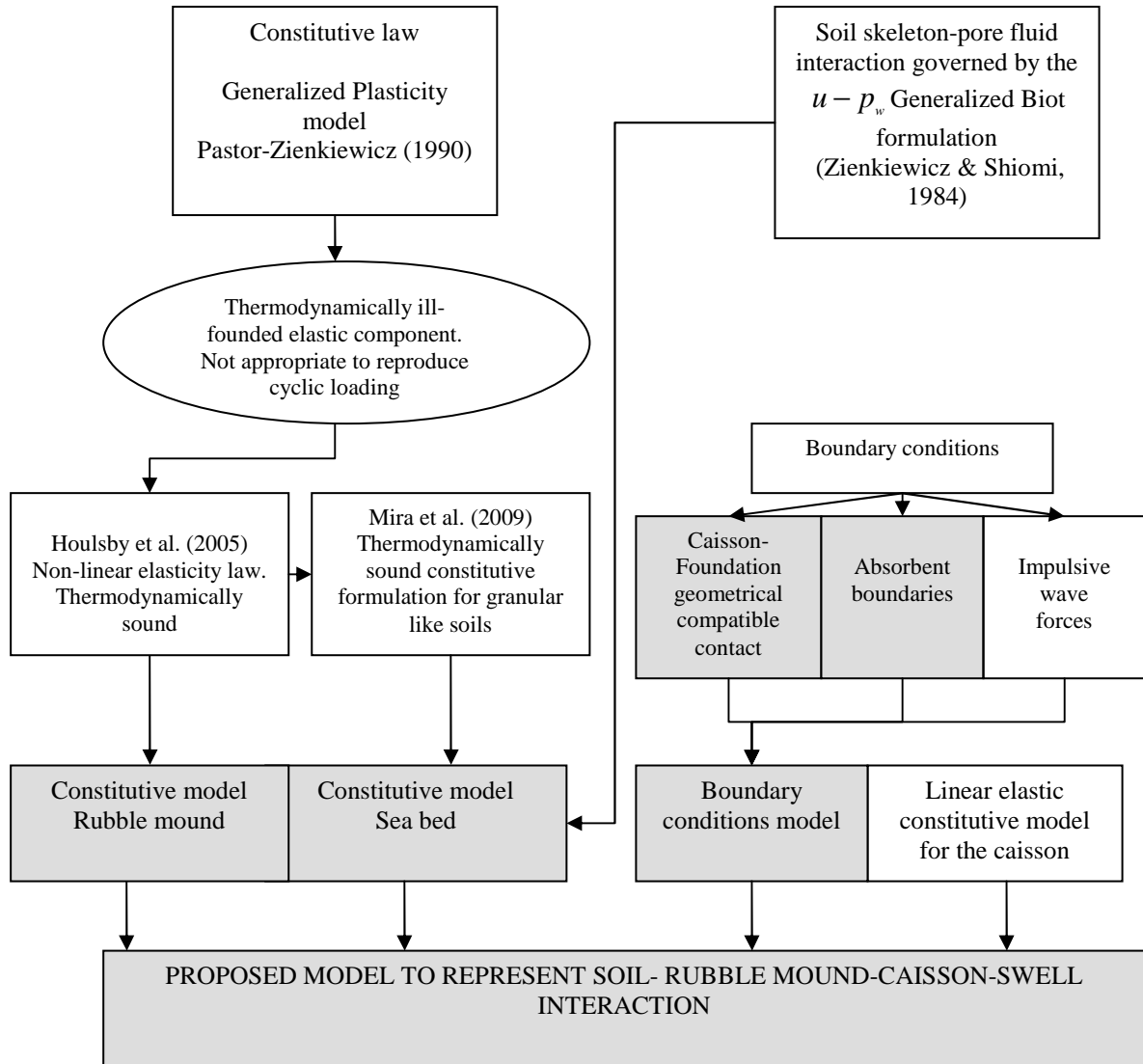


**Figure 1:** Physical systems involved in the soil-water-breakwater interaction model

The theoretical model for the soil-water-breakwater interaction proposed is developed in two dimensions under plain strain idealization.

Once the sea bed, rubble mound and caisson governing equations are derived, including the couplings involved as well as the initial and boundary conditions, the theoretical model for the soil-water-breakwater interaction proposed will be set.

In (Figure 2), the main parts of the theoretical model proposed in this paper in order to analyze the complex clay like-rubble mound-caisson-swell interaction are schematically shown. The novel theoretical contributions appear in this figure over a dark colour box.



**Figure 2:** Outline of the theoretical model proposed.

### 3 FINITE ELEMENT SOLUTION OF THE SETTLED GOVERNING EQUATIONS

Once the kinematic relations as well as the constitutive laws are integrated in the balance equations, a system of five partial differential equations with five field variables is established. The field variables involved are: sea bed skeleton displacement  $\mathbf{u}^{sb}$ , pore water pressure  $p_w^{sb}$ , rubble mound skeleton displacement  $\mathbf{u}^{rm}$ , pore water pressure  $p_w^{rm}$  and caisson displacement  $\mathbf{u}^{ca}$ .

The system of partial differential equations can be discretized using standard Galerkin techniques, as described in [11]. After spatial discretization,  $\mathbf{u}^{sb} \cong N^u \bar{\mathbf{u}}^{sb}$ ,  $p_w^{sb} \cong N^p \bar{p}_w^{sb}$ ,  $\mathbf{u}^{rm} \cong N^u \bar{\mathbf{u}}^{rm}$ ,  $p_w^{rm} \cong N^p \bar{p}_w^{rm}$ ,  $\mathbf{u}^{ca} \cong N^u \bar{\mathbf{u}}^{ca}$  the second order ordinary

differential equation system (1)-(3) is obtained

$$\mathbf{M}^{sb} \ddot{\mathbf{u}}^{sb} + \mathbf{C}^{sb} \dot{\mathbf{u}}^{sb} + \int_{\Omega^{sb}} \mathbf{B}^T \boldsymbol{\sigma}'^{sb} d\Omega^{sb} - \mathbf{Q}^{sb} \bar{\mathbf{p}}_w^{sb} - \mathbf{f}^{sb1} = \mathbf{0} \quad (1)$$

$$(\mathbf{Q}^{sb})^T \dot{\mathbf{u}}^{sb} + \mathbf{H}^{sb} \bar{\mathbf{p}}_w^{sb} + \mathbf{S}^{sb} \dot{\bar{\mathbf{p}}}_w^{sb} - \mathbf{f}^{sb2} = \mathbf{0}$$

$$\mathbf{M}^{rm} \ddot{\mathbf{u}}^{rm} + \mathbf{C}^{rm} \dot{\mathbf{u}}^{rm} + \int_{\Omega^{rm}} \mathbf{B}^T \boldsymbol{\sigma}'^{rm} d\Omega^{rm} - \mathbf{Q}^{rm} \bar{\mathbf{p}}_w^{rm} - \mathbf{f}^{rm1} = \mathbf{0} \quad (2)$$

$$(\mathbf{Q}^{rm})^T \dot{\mathbf{u}}^{rm} + \mathbf{H}^{rm} \bar{\mathbf{p}}_w^{rm} + \mathbf{S}^{rm} \dot{\bar{\mathbf{p}}}_w^{rm} - \mathbf{f}^{rm2} = \mathbf{0}$$

$$\mathbf{M}^{ca} \ddot{\mathbf{u}}^{ca} + \mathbf{C}^{ca} \dot{\mathbf{u}}^{ca} + \mathbf{K}^{ca} \bar{\mathbf{u}}^{ca} - \mathbf{f}^{ca} = \mathbf{0} \quad (3)$$

Where  $\mathbf{B} = \mathbf{S}\mathbf{N}^u$  and

$$\mathbf{f}^{sb1} = \int_{\Omega^{sb}} (\mathbf{N}^u)^T \rho^{sb} \mathbf{b} d\Omega^{sb} + \int_{\Gamma_t^{sb}} (\mathbf{N}^u)^T \mathbf{t}_{imp}^{sb} d\Gamma_t^{sb} + \mathbf{C}_r^{sb} \dot{\mathbf{u}}^{sb} \quad (4)$$

$$\mathbf{f}^{sb2} = \int_{\Omega^{sb}} (\nabla \mathbf{N}^p)^T \mathbf{k}^{sb} \rho_w^{sb} \mathbf{b} d\Omega^{sb} + \int_{\Gamma_{pw}^{sb}} (\mathbf{N}^p)^T \mathbf{q}_{imp}^{sb} d\Gamma_{pw}^{sb}$$

$$\mathbf{f}^{rm1} = \int_{\Omega^{rm}} (\mathbf{N}^u)^T \rho^{rm} \mathbf{b} d\Omega^{rm} + \int_{\Gamma_t^{rm}} (\mathbf{N}^u)^T \mathbf{t}_{imp}^{rm} d\Gamma_t^{rm} + \mathbf{C}_c^{rm} \dot{\mathbf{u}}^{rm} \quad (5)$$

$$\mathbf{f}^{rm2} = \int_{\Omega^{rm}} (\nabla \mathbf{N}^p)^T \mathbf{k}^{rm} \rho_w^{rm} \mathbf{b} d\Omega^{rm} + \int_{\Gamma_{pw}^{rm}} (\mathbf{N}^p)^T \mathbf{q}_{imp}^{rm} d\Gamma_{pw}^{rm}$$

$$\mathbf{f}^{ca} = \int_{\Omega^{ca}} (\mathbf{N}^u)^T \rho^{ca} \mathbf{b} d\Omega^{ca} + \int_{\Gamma_t^{ca}} (\mathbf{N}^u)^T \mathbf{t}_{imp}^{ca} d\Gamma_t^{ca} + \mathbf{C}_c^{ca} \dot{\mathbf{u}}^{ca} \quad (6)$$

The matrices given in the system (1)-(3) are defined by

$$\mathbf{M}^{sb} = \int_{\Omega^{sb}} (\mathbf{N}^u)^T \rho^{sb} \mathbf{N}^u d\Omega^{sb}, \mathbf{M}^{rm} = \int_{\Omega^{rm}} (\mathbf{N}^u)^T \rho^{rm} \mathbf{N}^u d\Omega^{rm}, \mathbf{M}^{ca} = \int_{\Omega^{ca}} (\mathbf{N}^u)^T \rho^{ca} \mathbf{N}^u d\Omega^{ca} \quad (7)$$

$$\mathbf{Q}^{sb} = \int_{\Omega^{sb}} \mathbf{B}^T \mathbf{m} \mathbf{N}^p d\Omega^{sb}, \mathbf{Q}^{rm} = \int_{\Omega^{rm}} \mathbf{B}^T \mathbf{m} \mathbf{N}^p d\Omega^{rm} \quad (8)$$

$$\mathbf{S}^{sb} = \int_{\Omega^{sb}} (\mathbf{N}^p)^T \frac{1}{Q^{sb}} (\mathbf{N}^p) d\Omega^{sb}, \mathbf{S}^{rm} = \int_{\Omega^{rm}} (\mathbf{N}^p)^T \frac{1}{Q^{rm}} (\mathbf{N}^p) d\Omega^{rm} \quad (9)$$

$$\mathbf{H}^{sb} = \int_{\Omega^{sb}} (\nabla \mathbf{N}^p)^T \frac{\mathbf{k}^{sb}}{\rho_w \cdot g} (\nabla \mathbf{N}^p) d\Omega^{sb}, \mathbf{H}^{rm} = \int_{\Omega^{rm}} (\nabla \mathbf{N}^p)^T \frac{\mathbf{k}^{rm}}{\rho_w \cdot g} (\nabla \mathbf{N}^p) d\Omega^{rm} \quad (10)$$

$$\mathbf{C}^{sb} = \alpha^{sb} \mathbf{M}^{sb} + \beta^{sb} \mathbf{K}^{sb}, \mathbf{C}^{rm} = \alpha^{rm} \mathbf{M}^{rm} + \beta^{rm} \mathbf{K}^{rm}, \mathbf{C}^{ca} = \alpha^{ca} \mathbf{M}^{ca} + \beta^{ca} \mathbf{K}^{ca} \quad (11)$$

$$\mathbf{K}^{sb} = \int_{\Omega^{sb}} \mathbf{B}^T \mathbf{D}^{sb} (\boldsymbol{\sigma}'^{sb}) \mathbf{B} d\Omega^{sb}, \mathbf{K}^{rm} = \int_{\Omega^{rm}} \mathbf{B}^T \mathbf{D}^{rm} (\boldsymbol{\sigma}'^{rm}) \mathbf{B} d\Omega^{rm}, \mathbf{K}^{ca} = \int_{\Omega^{ca}} \mathbf{B}^T \mathbf{D}^{ca} \mathbf{B} d\Omega^{ca} \quad (12)$$

The  $\mathbf{C}_r^{sb}$  term in (4) represents the contribution of the radiation boundaries to the discretized governing equations, while  $\mathbf{C}_c^{rm}$  and  $\mathbf{C}_c^{ca}$  terms, appearing in (5) and (6), respectively, represents the contribution of the rubble mound - caisson contact to the discretized governing equations.

The proper choice of the element type in order to discretize the computational domain is of paramount importance as some elements introduce errors leading to unrealistic limit loads and spurious failure elements [12]. Under Babuska-Brezzi robustness condition, keeping in mind the need of a  $C^0$  interpolation for each field variable, in the present paper a mixed isoparametric lagrangian triangular element has been used, with 6 nodes quadratic interpolation for any skeleton displacement,  $\mathbf{u}^{sb}$ ,  $\mathbf{u}^{rm}$  y  $\mathbf{u}^{ca}$  and 3 node linear interpolation for pore water pressure interpolation,  $p_w^{sb}$ ,  $p_w^{rm}$ .

Temporal discretization of the displacements  $\bar{\mathbf{u}}^{global} = [\bar{\mathbf{u}}^{sb}, \bar{\mathbf{u}}^{rm}, \bar{\mathbf{u}}^{ca}]^T$  is performed by the Generalized Newmark GN22 scheme while the excess pore pressure of the sea bed and rubble mound  $\bar{\mathbf{p}}_w^{global} = [\bar{\mathbf{p}}_w^{sb}, \bar{\mathbf{p}}_w^{rm}]^T$  are discretized by the GN11 scheme, leading to the following difference equation systems

$$\begin{aligned} \ddot{\bar{\mathbf{u}}}_{n+1}^{global} &= \ddot{\bar{\mathbf{u}}}_n^{global} + \Delta \ddot{\bar{\mathbf{u}}}_n^{global} \\ \dot{\bar{\mathbf{u}}}_{n+1}^{global} &= \dot{\bar{\mathbf{u}}}_n^{global} + \Delta t \cdot \ddot{\bar{\mathbf{u}}}_n^{global} + \beta_1 \cdot \Delta t \cdot \Delta \ddot{\bar{\mathbf{u}}}_n^{global} \\ \bar{\mathbf{u}}_{n+1}^{global} &= \bar{\mathbf{u}}_n^{global} + \Delta t \cdot \dot{\bar{\mathbf{u}}}_n^{global} + \frac{1}{2} \Delta t^2 \cdot \ddot{\bar{\mathbf{u}}}_n^{global} + \frac{1}{2} \Delta t^2 \cdot \beta_2 \cdot \Delta \ddot{\bar{\mathbf{u}}}_n^{global} \end{aligned} \quad (13)$$

$$\begin{aligned} \dot{\bar{\mathbf{p}}}_{wn+1}^{global} &= \dot{\bar{\mathbf{p}}}_{wn}^{global} + \Delta \dot{\bar{\mathbf{p}}}_{wn}^{global} \\ \bar{\mathbf{p}}_{wn+1}^{global} &= \bar{\mathbf{p}}_{wn}^{global} + \Delta t \cdot \dot{\bar{\mathbf{p}}}_{wn}^{global} + \Delta t \cdot \beta_1 \cdot \Delta \dot{\bar{\mathbf{p}}}_{wn}^{global} \end{aligned} \quad (14)$$

After the incorporation of difference equation (13) and (14) in (1)-(3) a non linear algebraic system is obtained where the unknown values are  $[\Delta \ddot{\bar{\mathbf{u}}}_n^{sb}, \Delta \dot{\bar{\mathbf{p}}}_{wn}^{sb}, \Delta \ddot{\bar{\mathbf{u}}}_n^{rm}, \Delta \dot{\bar{\mathbf{p}}}_{wn}^{rm}, \Delta \ddot{\bar{\mathbf{u}}}_n^{ca}]$ . The Newton-Raphson scheme is used to solved the non linear algebraic in each time step, obtaining the values of the displacements  $\bar{\mathbf{u}}_{n+1}^{global}$  and pore water pressure  $\bar{\mathbf{p}}_{wn+1}^{global}$  at time  $t_{n+1}$  by the difference equations (13) and (14).

#### 4 ADÍNDICA PROGRAM.

The soil-water-structure interaction involved in a breakwater structure subjected to sea wave actions is not restricted to a unique engineering discipline. Taking into account the limitations of commercial codes to deal with this kind of multidisciplinary phenomena, the authors of the present paper have decided to fully develop the numerical solution of the settled governing equations. Furthermore, a program called ADÍNDICA has been created in M Matlab language. ADÍNDICA is a Spanish acronym for “Caisson Breakwater Dynamic Analysis”.

This program can be used to design gravity maritime structures foundations. The users of ADÍNDICA program may be able to analyze the fundamental aspects involved in the geomechanic behaviour associated with the foundation of this kind of structures. These aspects are: *i*) the complex caisson-rubble mound interaction derived from the swell dynamic and cyclic action, allowing an estimation of the stresses transmitted to the seabed, *ii*) the soil skeleton-pore fluid coupling, essential to estimate the pore pressure variation influenced by the elastic compression of the pore fluid (air), by elastic compression and dilatation of the soil skeleton in combination with limited drainage, and *iii*) the possible change of soil strength and stiffness with time mainly due to repetitive loading and /or consolidation, allowing a degradation estimation of the seabed and long term effects. The principal features of ADÍNDICA code [13] are the following:

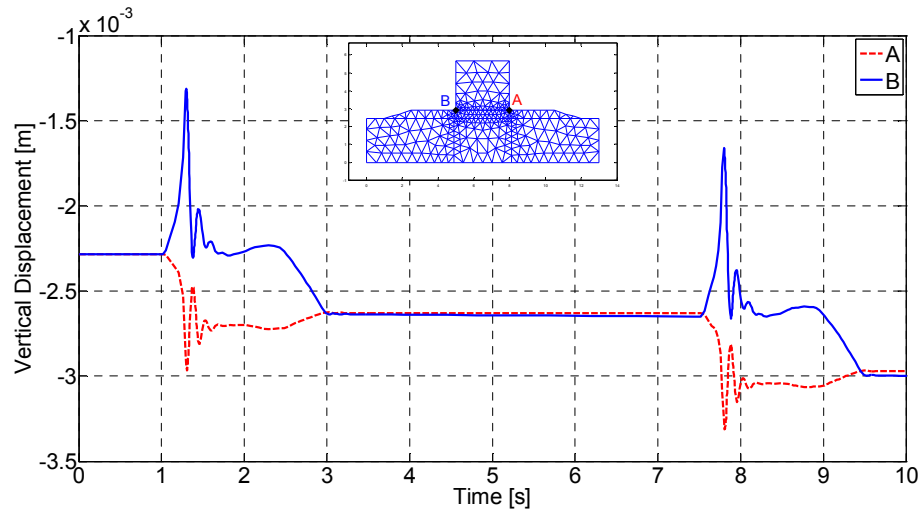
- Two dimensional finite element program under plane strain and axisymmetric idealization.
- Pre and Postprocessor without leaving Matlab environment. It is also coupled with GID for specific situations.
- Static, consolidation and dynamic problems can be solved.
- Linear and nonlinear problems (material and contact-impact) can be solved.
- Robust and accurate local integration algorithm for elastoplastic constitutive law as well as global load-displacement integration algorithm.
- Point and distributed loads can be defined on the boundary. Both in static and dynamic fashion.
- Stage constructions can be designed

#### 5 RESULTS AND CONCLUSIONS

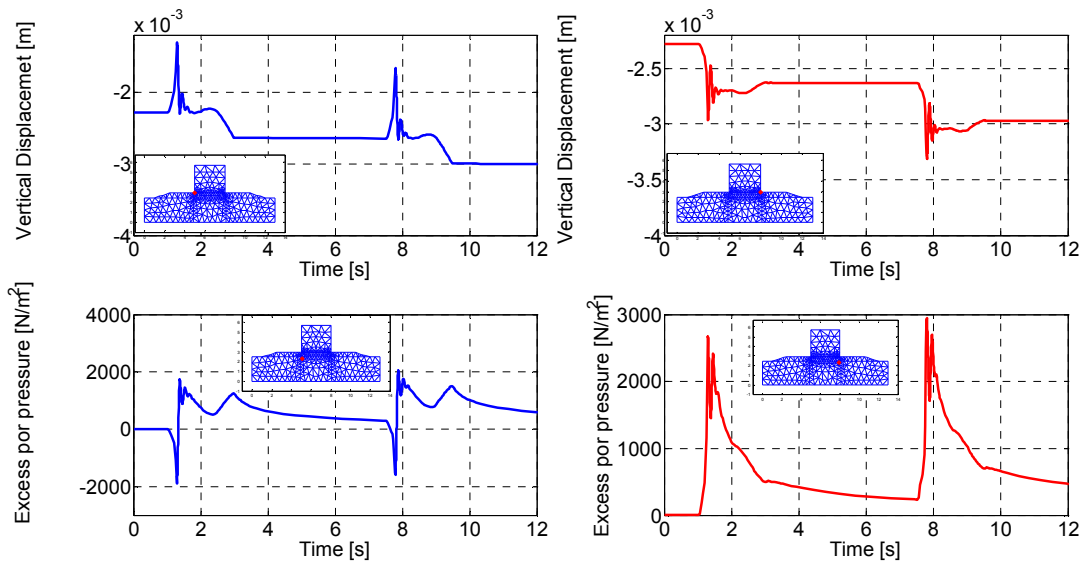
The large scale model conducted in 2004 by Kudella and Oumeraci [14] in the Large Wave Flume (GWK) of Hannover is numerically reproduced under the scope of the soil-water-structure interaction model proposed in the present paper.

ADÍNDICA code has been able to reproduce adequately the principal characteristics of the caisson oscillations and instantaneous pore pressure generation experimentally deduced (Figure 3, Figure 4, Figure 5). These characteristics are: *i*) the magnitude of the caisson motions induced by impact load at the seaward edge is higher than at the shoreward edge, *ii*) The shape of the  $p_w$  records closely follow the shape of the vertical movement records of the caisson edges with the reverse sign. Indeed, the relatively small downward caisson motion at shoreward edge induces a much higher positive pore pressure amplitude than the negative amplitude at seaward edge, where the upward motion is higher and *iii*) The influence of

caisson motions on pore pressure generation decreases with increasing depth.

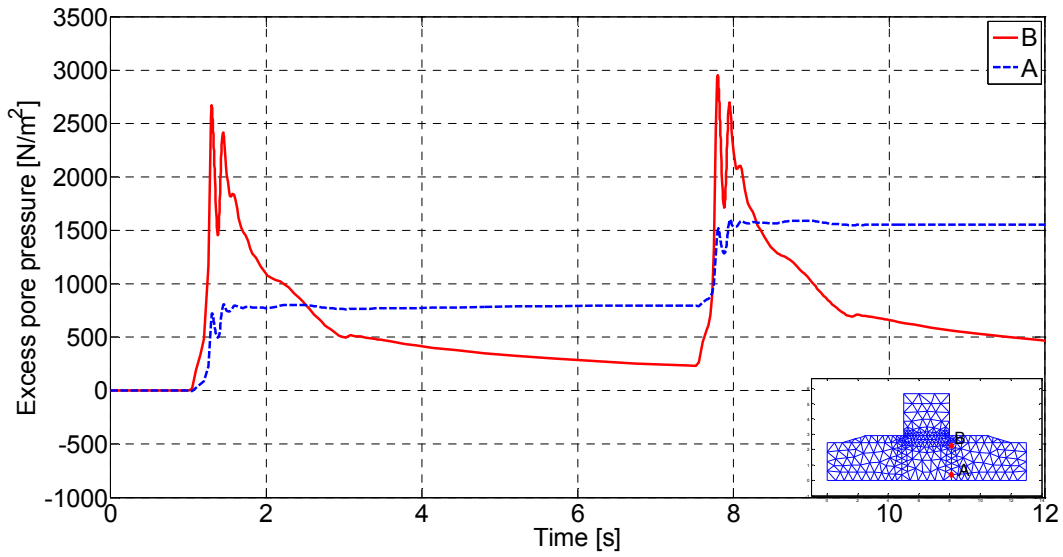


**Figure 3:** Vertical displacement obtain at caisson edges induced by two impulsive wave actions ( $H = 0.6m$ ,  $T = 6.5s$ ,  $h_s = 1.6m$ ,  $h_l = 0.6m$ ). ADÍNDICA numerical result.



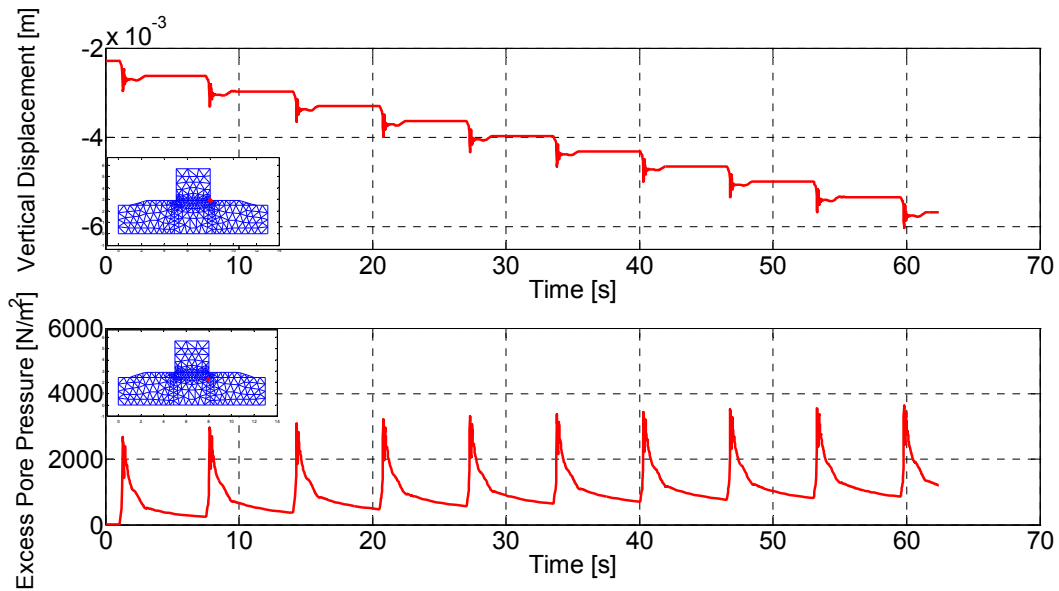
**Figure 4:** Caisson edges vertical displacement excess pore pressure relation ( $H = 0.6m$ ,  $T = 6.5s$ ,  $h_s = 1.6m$ ,  $h_l = 0.6m$ ). ADÍNDICA numerical result.





**Figure 5:** Excess pore pressures obtained at different heights in the sand layer ( $H = 0.6m$ ,  $T = 6.5s$ ,  $h_s = 1.6m$ ,  $h_l = 0.6m$ ). ADÍNDICA numerical result.

ADÍNDICA code is able to reproduce satisfactorily the accumulative settlement behaviour of a vertical breakwater structure subjected to series of sea wave impacts. It is also able to simulate adequately the correlation between accumulated settlements and residual pore pressure (Figure 6).



**Figure 6:** Relation between accumulated settlement and residual pore pressure ( $H = 0.6m$ ,  $T = 6.5s$ ,  $h_s = 1.6m$ ,  $h_l = 0.6m$ ). ADÍNDICA numerical result.

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